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Spin-splitting enhanced by many-body effects in a two-dimensional electron gas in the presence of the Rashba spin–orbit interaction

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Abstract

We present a theoretical study on how many-body effects, especially the exchange interaction, can affect the spin-splitting of a two-dimensional electron gas (2DEG) in the presence of the Rashba spin–orbit interaction. The standard Hartree–Fock approximation and Green's function approach are employed to calculate self-consistently the energy spectrum and density of states of a spin-split 2DEG realized from an InGaAs/InAlAs heterojunction. We find that although the Hartree interaction affects rather weakly the self-energy of a quasi-particle, the presence of the exchange (Fock) interaction can significantly enhance the spin-splitting of a 2DEG on top of the Rashba effect. The main physical reasons for this important phenomenon are examined and discussed.

1. Introduction

At present, one important aspect in the field of spin electronics (or spintronics) is to study electronic systems with a finite spin-splitting realized from narrow-gap semiconductor quantum wells in the absence of an external magnetic field. It is known that in such systems the spontaneous spin-splitting of the carriers can be achieved at zero magnetic field through the inversion asymmetry of the microscopic confining potential owing to the presence of the heterojunction [1, 2]. This corresponds to an inhomogeneous surface electric field and, hence, is electrically equivalent to the Rashba spin-splitting or Rashba effect [3]. Due to the unique features of the spin–orbit interaction (SOI) and to the important and interesting consequences of such interaction, the Rashba spintronic systems have been proposed as novel electronic devices such as spin-transistors [4], spin-filters [5], spin-waveguides [6], etc, which

have potential applications in quantum computation and communication. In recent years, the investigation of the Rashba spintronic systems has become an important and fast-growing research area in condensed matter physics and semiconductor electronics, with large amounts of theoretical [7–9] and experimental [10–14] work being published.

Experimentally, the Rashba spin-splitting in a narrow-gap quantum well can be identified simply through magneto-transport measurements under which the Shubnikov–de Haas oscillation can be observable [10–14]. Theoretically, it was shown that the effects of the Rashba spin-splitting in narrow-gap semiconductor quantum wells (e.g. InGaAs/InAlAs based) can be studied simply by using the $\mathbf{k}\cdot\mathbf{p}$ band-structure calculation [1, 2] and by self-consistent calculations on the basis of the $\mathbf{k}\cdot\mathbf{p}$ results [9]. It should be pointed out that there is a poor agreement between the theoretically estimated Rashba parameter and the experimentally measured value [15]. The published experimental results have indicated that in InGaAs/InAlAs-based two-dimensional electron gas (2DEG) systems, the value of the Rashba parameter can reach up to (3–4) $\times$ 10^{-11} eV m [7, 16], and can even be higher, to 8 $\times$ 10^{-11} eV m [12]. These experimental data are much larger (by at least a factor of two) than those obtained from theoretical calculations such as the $\mathbf{k}\cdot\mathbf{p}$ band-structure calculation [2] and self-consistent calculation based on the $\mathbf{k}\cdot\mathbf{p}$ results [9]. In order to explore the intrinsic origin of a strong spontaneous spin splitting in narrow-gap semiconductor-based 2DEGs, other different theoretical models have been developed. The envelope-function theory showed that the origin of the large value of the Rashba parameter may come from the 'interface contribution' [8, 10, 11]. However, to the best of our knowledge, up to now a tractable theoretical approach to calculate the 'interface contribution' to the Rashba parameter has not been reported yet. In addition, it is suggested that a difference of the electron effective masses in various layers of a heterostructure can also result in an additional force and so enlarge the Rashba parameter [17].

It should be noted that the above-mentioned theoretical work has mainly focused on single-particle aspects of the spin-split 2DEG systems. For example, the $\mathbf{k}\cdot\mathbf{p}$ calculation is basically a single-particle approach. Although the Hartree potential induced by charge distribution along the growth direction has been taken into account, the self-consistent calculation [9] normally cannot give a full consideration of the many-body effects along the 2D plane. Hence, it is of significance and importance to look into how many-body effects play roles in affecting the spin-splitting of a 2DEG in the presence of the Rashba effect, and this becomes the prime motivation of the present study. It is known that, in sharp contrast to a spin-degenerated 2DEG in which the spin-splitting (e.g. Zeeman splitting) is achieved only in the energy space, the Rashba spin-splitting depends strongly on the wavevector (or momentum) of an electron. Because the electron–electron (e–e) interaction via Coulomb potential depends explicitly on electron wavevector and the interaction can be achieved via altering the electron wavevector as well, it is expected that the e–e interaction can affect significantly the Rashba spin-splitting. The previous theoretical work on the basis of a Fermi liquid theory [18] has shown that many-body effects such as exchange interaction can enhance the spin–orbit interaction (SOI) in a 2DEG system. In this paper, we develop a simpler and more tractable theoretical approach to demonstrate that the e–e interaction does strongly affect the spin-splitting in a 2DEG on top of the Rashba effect. We employ the standard Hartree–Fock approximation and Green's function approach to examine how the many-body effects can influence the spin-splitting of the Rashba spintronic systems. In contrast to what has been obtained by Chen and a co-worker [18], here we intend to study the effect of e–e interaction on other spintronic properties which are experimentally measurable. Our theoretical approaches are developed and presented in section 2 and the results obtained from this study are discussed and analysed in section 3. The main conclusions drawn from this work are summarized in section 4.

2. Theoretical approaches

The exchange-enhanced spin-splitting in a 2DEG in high magnetic fields in the absence of SOI has been well studied theoretically in the past [19–21]. In the present study we generalize these well developed theoretical approach to the investigation of the exchange interaction in a 2DEG in the presence of the Rashba SOI. We consider a typical 2DEG formed in the xy-plane (taken as the 2D-plane) in a narrow-gap semiconductor quantum well, such as an InGaAs/InAlAs heterojunction in which the growth direction is taken along the z-axis. In such a case, the effect of SOI can be obtained from, e.g., a $\mathbf{k}\cdot\mathbf{p}$ band-structure calculation [16]. Including the lowest order of the SOI induced by the Rashba effect, the single-electron Hamiltonian can be solved analytically [6]. The electron wavefunction and the corresponding energy spectrum are given, respectively, as

$$\Psi_\nu(\mathbf{R}) = \mathcal{C}[1, -\mathrm{i}\sigma \mathrm{e}^{-\mathrm{i}\phi}]\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\psi_n(z) \tag{1}$$

being in the form of a row matrix, and

$$E_{\sigma n}(\mathbf{k}) = E_\sigma(\mathbf{k}) + \varepsilon_n = \frac{\hbar^2 k^2}{2m^*} + \sigma\alpha k + \varepsilon_n. \tag{2}$$

Here, $\mathcal{C} = 1/\sqrt{2}$, $\nu = (\sigma \mathbf{k} n)$ represents all quantum numbers, $\mathbf{k} = (k_x, k_y)$ is the electron wavevector along the 2D plane, $\mathbf{R} = (\mathbf{r}, z) = (x, y, z)$, m^* is the electron effective mass, ϕ is an angle between $\mathbf{k}$ and the x-axis, α is the Rashba parameter or spin–orbit coupling parameter, which measures the strength of the SOI, and $\sigma = \pm 1$ refers to different spin branches in $\mathbf{k}$-space. In equations (1) and (2), the wavefunction $\psi_n(z)$ and energy ε_n for an electron in the nth electronic subband are determined by a spin-independent Schrödinger equation along the growth direction, because SOI does not affect the electronic states along the z-direction. From these results, one can immediately see that in the presence of the Rashba SOI (i) the electronic states are split into two spin branches in $\mathbf{k}$-space and electrons are oriented perpendicular to the electronic momentum in the 2D plane, (ii) the energy dispersion of a 2DEG is not parabolic due to the inclusion of the SOI term, and (iii) the energy levels for the $\pm$ spin branches depend strongly on electron wavevector (or momentum), where the energy separation between the two spin channels is a linear-in-k term. These features are in sharp contrast to those for a spin-degenerate 2DEG.

In the present work, we limit ourselves to the study of the many-body effect of a spin-split 2DEG, such as spin self-energy induced by electron–electron (e–e) interaction under a standard Hartree–Fock approximation [22]. Using the single-electron wavefunction given by equation (1), the two-particle Slater wavefunctions can be formed through

$$\Psi_{\nu\nu_1}(\mathbf{R}_1, \mathbf{R}_2) = \mathcal{C}[\Psi_\nu(\mathbf{R}_1)\Psi_{\nu_1}(\mathbf{R}_2) - \Psi_{\nu_1}(\mathbf{R}_1)\Psi_\nu(\mathbf{R}_2)]$$

and

$$\Psi^*_{\nu'\nu'_1}(\mathbf{R}_1, \mathbf{R}_2) = \mathcal{C}[\Psi^*_{\nu'}(\mathbf{R}_1)\Psi^*_{\nu'_1}(\mathbf{R}_2) - \Psi^*_{\nu'_1}(\mathbf{R}_1)\Psi^*_{\nu'}(\mathbf{R}_2)].$$

The electrostatic scattering energy induced by e–e interaction via the Coulomb potential $V(\mathbf{R}) = e^2/(\kappa|\mathbf{R}|)$ becomes

$$E_{\mathrm{ee}} = \langle\Psi^*_{\nu'\nu'_1}(\mathbf{R}_1, \mathbf{R}_2)|V(\mathbf{R}_1 - \mathbf{R}_2)|\Psi_{\nu\nu_1}(\mathbf{R}_1, \mathbf{R}_2)\rangle = V^{\mathrm{H}}_{\nu'\nu\nu'_1\nu_1} - V^{\mathrm{F}}_{\nu'_1\nu\nu'\nu_1}. \tag{3}$$

Here, the first (second) term is induced by forward or Hartree (backward or exchange) scattering, and

$$V^{\mathrm{H}}_{\nu'\nu\nu'_1\nu_1} = \delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}\delta_{\mathbf{k}'_1,\mathbf{k}_1-\mathbf{q}}(V_q/4)F_{n'nn'_1n_1}(q)(1 + \sigma'\sigma\mathrm{e}^{\mathrm{i}(\phi-\phi')})(1 + \sigma'_1\sigma_1\mathrm{e}^{\mathrm{i}(\phi_1-\phi'_1)})$$

and

$$V^{\mathrm{F}}_{\nu'_1\nu\nu'\nu_1} = \delta_{\mathbf{k}'_1,\mathbf{k}+\mathbf{q}}\delta_{\mathbf{k}',\mathbf{k}_1-\mathbf{q}}(V_q/4)F_{n'_1nn'n_1}(q)(1 + \sigma'_1\sigma\mathrm{e}^{\mathrm{i}(\phi-\phi'_1)})(1 + \sigma'\sigma_1\mathrm{e}^{\mathrm{i}(\phi_1-\phi')}),$$

where $V_q = 2\pi e^2/\kappa q$, $\mathbf{q} = (q_x, q_y)$ is the change of electron wavevector along the 2D plane during an e–e scattering event, κ is the dielectric constant of the material, and $F_{n'nn'_1n_1}(q) = \int \mathrm{d}z_1 \int \mathrm{d}z_2 \ \psi^*_{n'}(z_1)\psi_n(z_1)\mathrm{e}^{-q|z_1-z_2|}\psi^*_{n'_1}(z_2)\psi_{n_1}(z_2)$ is the form factor for e–e scattering in a 2DEG. Moreover, the linear-momentum conservation laws have been included within the Hartree and exchange terms. Due to the overlap of electron wavefunctions in different spin channels, the scattering energies for Hartree and exchange interactions differ from those in the absence of the SOI [22].

From now on, we consider a strongly confined 2DEG in an InGaAs/InAlAs heterojunction. In such a case, only the lowest electronic subband is occupied by an electron and we take $n' = n = 0$ and measure the energy from $\varepsilon_0 = 0$. Furthermore, we neglect the penetration of the electron wavefunction along the z-direction so that $F_{0000}(q) \simeq 1$. For convenience of using the notations, we define $\beta = (\sigma, \sigma')$ for electronic transition from a spin branch σ' to a branch σ, where $\beta = 1 = (+,+)$, $\beta = 2 = (+,-)$, $\beta = 3 = (-,+)$, and $\beta = 4 = (-,-)$ for four electronic transition channels. Here, for intra-SO transitions $\beta = 1$ or 4 and the product $\sigma\sigma' = +1$, whereas for inter-SO transitions $\beta = 2$ or 3 and the product $\sigma\sigma' = -1$. After taking into consideration that the linear-momentum that flows into the e–e scattering conserves with what flows out for different scattering processes, we obtain

$$V^{\mathrm{H}}_{\nu'\nu\nu'_1\nu_1} = V^{\mathrm{H}}_{\beta\gamma} \simeq \lim_{q\to 0} V_q \delta_{\sigma',\sigma}\delta_{\sigma'_1,\sigma_1},$$

and

$$V^{\mathrm{F}}_{\nu'_1\nu\nu'\nu_1} = V^{\mathrm{F}}_{\beta\gamma} \simeq \delta_{\mathbf{k}'_1,\mathbf{k}+\mathbf{q}}\delta_{\mathbf{k}',\mathbf{k}_1-\mathbf{q}}(V_q/4)(1+\sigma'_1\sigma\mathrm{e}^{\mathrm{i}\theta})(1+\sigma'\sigma_1\mathrm{e}^{\mathrm{i}\theta}),$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{k}'_1$. We note that for a spin-split 2DEG, because the spin-splitting depends explicitly on electron wavevector, the requirement of the linear-momentum conservation can lead to a consequence that the electron spin may flip during an e–e scattering event, especially for the exchange term.

From the electron energy spectrum given by equation (2), we can derive the retarded and advanced Green's functions for the electron. Applying these Green's functions along with the bare e–e interaction to the diagrammatic techniques to derive the effective e–e interaction under the random phase approximation (RPA), we can obtain the effective e–e interaction. Thus, the dielectric function matrix can be obtained as [23]

$$\epsilon(\Omega, q) = \begin{bmatrix} 1+a_1 & 0 & 0 & a_4 \\ 0 & 1+a_2 & a_3 & 0 \\ 0 & a_2 & 1+a_3 & 0 \\ a_1 & 0 & 0 & 1+a_4 \end{bmatrix}. \tag{4}$$

Here, $a_\beta = -V_q G_0(q)\Pi_\beta(\Omega, q)$ with $G_0(q) = \int \mathrm{d}z_1 \int \mathrm{d}z_2 \ |\psi_0(z_1)|^2|\psi_0(z_2)|^2\mathrm{e}^{-q|z_1-z_2|}$ and

$$\Pi_{\sigma'\sigma}(\Omega, q) = \sum_{\mathbf{k}} A^{\sigma'\sigma}_{\mathbf{kq}} \frac{f[E_{\sigma'}(\mathbf{k}+\mathbf{q})] - f[E_\sigma(\mathbf{k})]}{\hbar\Omega + E_{\sigma'}(\mathbf{k}+\mathbf{q}) - E_\sigma(\mathbf{k}) + \mathrm{i}\delta}$$

is the pair bubble or density–density correlation function in the absence of e–e interaction [24, 25], where $f(x)$ is the Fermi–Dirac function, an infinitesimal quantity $\mathrm{i}\delta$ has been introduced to make the integral converge when Fourier transforming from time-representation to spectrum-representation, $A^{\sigma'\sigma}_{\mathbf{kq}} = [1 + \sigma'\sigma(k + q\cos\psi)/|\mathbf{k}+\mathbf{q}|]/2$ is a spin-dependent factor, and ψ is an angle between $\mathbf{k}$ and $\mathbf{q}$. Moreover, the inverse dielectric function matrix for a spin-split 2DEG is

$$\epsilon^{-1}(\Omega, q) = \begin{bmatrix} 1-a^*_1 & 0 & 0 & -a^*_4 \\ 0 & 1-a^*_2 & -a^*_3 & 0 \\ 0 & -a^*_2 & 1-a^*_3 & 0 \\ -a^*_1 & 0 & 0 & 1-a^*_4 \end{bmatrix}, \tag{5}$$

with $a_1^* = a_1/(1+a_1+a_4)$, $a_2^* = a_2/(1+a_2+a_3)$, $a_3^* = a_3/(1+a_2+a_3)$, and $a_4^* = a_4/(1+a_1+a_4)$. It should be noted that in contrast to equation (15) in [24] we use a matrix to present the dielectric function $\epsilon(\Omega, q)$. For a spin-split 2DEG which is basically a two-level system when only the lowest subband is considered, there are four channels for electronic transitions (i.e. $\beta = 1, 2, 3$, and 4 defined here) induced by e–e interaction. From the fact that a transition event β should be affected by other transition events due to e–e interaction, the dielectric function or the inverse dielectric function for a spin-split 2DEG is therefore a 4×4 matrix. In the presence of the e–e screening and with the inverse dielectric function matrix, the effective scattering energy for Hartree and Fock interactions can be calculated respectively by

$$\mathcal{V}^{\mathrm{H}}_{\beta\gamma} = V^{\mathrm{H}}_{\beta\gamma}\epsilon^{-1}_{\gamma\beta}(q) \qquad \text{and} \qquad \mathcal{V}^{\mathrm{F}}_{\beta\gamma} = V^{\mathrm{F}}_{\beta\gamma}\epsilon^{-1}_{\gamma\beta}(q), \tag{6}$$

with $\epsilon_{\beta\gamma}(q) = \lim_{\Omega\to0}\epsilon_{\beta\gamma}(\Omega, q)$ being the element of the static dielectric function matrix. Thus, in the presence of e–e screening, the self-energy induced by e–e interaction at a low-temperature (i.e., $T \to 0$) can be calculated through

$$\Sigma^{\sigma'}_{\sigma}(k) = \sum_{\mathbf{k}'<\mathbf{k}_{\mathrm{F}}^{\sigma'}}\sum_{\sigma_1',\sigma_1}[\mathcal{V}^{\mathrm{H}}_{\beta\gamma} - \mathcal{V}^{\mathrm{F}}_{\beta\gamma}] = \sum_{\mathbf{k}'<\mathbf{k}_{\mathrm{F}}^{\sigma'}}[\lim_{q\to0}2\delta_{\sigma',\sigma} - \delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}]\frac{2\pi e^2}{\kappa[q + K_{\sigma'\sigma}(q)]}, \tag{7}$$

where $\mathbf{k}_{\mathrm{F}}^{\sigma}$ is the Fermi wavevector for the σ spin branch and $K_{\sigma'\sigma}(q)$ is the inverse screening length caused by electronic transition in difference spin channels. Equation (7) indicates that for a spin-split 2DEG, the Hartree interaction does not affect the self-energy for inter-SO transitions. It has been shown that the inverse RPA screening length induced by intra-SO transition $K_{\sigma\sigma}(q) \to +\infty$ when $q \to 0$ [26]. As a result, in the presence of e–e screening, the Hartree interaction does not contribute to self-energy for intra-SO transition either. This result is similar to a spin-degenerate electron gas system in which the Hartree term vanishs by e–e screening via, e.g., the jellium model or the RPA approach [27]. Therefore, for a 2DEG in the presence of the Rashba effect, the self-energy comes mainly from exchange interaction, which reads

$$\Sigma^{\sigma'}_{\sigma}(k) = -\sum_{\mathbf{k}'<\mathbf{k}_{\mathrm{F}}^{\sigma'}}\frac{2\pi e^2}{\kappa[q + K_{\sigma'\sigma}(q)]}\delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}. \tag{8}$$

Similar to a spin-degenerate electron gas system [22], the self-energy for a spin-split 2DEG is negative, which implies that the e–e interaction lowers the energy of the system as one can expect. The inverse RPA screening length has been obtained in a recent theoretical work [26]. For intra- ($\sigma'\sigma = +$) and inter-SO ($\sigma'\sigma = -$) transition, we have

$$K_{\pm}(q) = \frac{16e^2m^*}{\pi\hbar^2\kappa q}\sum_{\sigma}\int_0^{\infty}\mathrm{d}k\,H^{\pm}_{\sigma}(k, q)\frac{f[E_{\sigma}(k)]k(k+q)}{(2k+q+2\sigma k_{\alpha})(k+q-|k-q|)} \tag{9}$$

where n_{σ} is the electron density in the σ spin branch, $k_{\alpha} = \alpha m^*/\hbar^2$, and

$$H^{\pm}_{\sigma}(k, q) = \frac{-1\pm1}{2}K(\mathcal{A}) + \Pi(\mathcal{A}B_{\pm}, \mathcal{A}) + \frac{q(q+2\sigma k_{\alpha})}{4k(k+\sigma k_{\alpha})}[\Pi(\mathcal{A}C_{\pm}, \mathcal{A}) - \Pi(\mathcal{A}B_{\pm}, \mathcal{A})]$$

with $K(x)$ and $\Pi(n, x) = \Pi(\pi/2, n, x)$ being respectively the complete elliptic integrals of the first and third kinds, $\mathcal{A} = (k+q-|k-q|)/(k+q+|k-q|)$, $\mathcal{B}_{\pm} = [(2k+q)/q]^{\pm1}$, and $\mathcal{C}_{\pm} = [(q-2\sigma k_{\alpha})/(2k+q+2\sigma k_{\alpha})]^{\pm1}$. These results can result in the fact that for a spin-split 2DEG the self-energy differs for different transition channels because of different screening lengths.

Applying the self-energy induced by e–e interaction to the diagrammatic techniques, the Green's function for a spin-split 2DEG can be represented by

$$G_{\sigma\sigma'}(E, k) = [G^{-1}_{\sigma}(E, k) - \Sigma^{\sigma'}_{\sigma}(k)]^{-1} \tag{10}$$

where $G_\sigma(E,k)=[E-E_\sigma(k)+\mathrm{i}\delta]^{-1}$ is the diagonal Green's function for a spin-split 2DEG in the absence of e–e interaction, and $E_\sigma(k)=\hbar^2k^2/2m^*+\sigma\alpha k$ is the energy spectrum without e–e scattering. In the form of a matrix, we have

$$G_{\sigma\sigma'}(E,k)=\frac{1}{\Delta_0+\mathrm{i}\Delta_1\delta}\begin{bmatrix}E_-+\mathrm{i}\delta & -\Sigma_+^-(k)\\ -\Sigma_-^+(k) & E_++\mathrm{i}\delta\end{bmatrix}. \tag{11}$$

Here $E_\sigma=E-E_\sigma(k)-\Sigma_\sigma^\sigma(k)$, $\Delta_0=E_+E_--\Sigma_+^-(k)\Sigma_-^+(k)$, and $\Delta_1=E_++E_-$. The corresponding electron density of states (DOS) is then given by

$$D_{\sigma\sigma'}(E)=-\frac{1}{\pi}\sum_{\mathbf{k}}\mathrm{Im}\,G_{\sigma\sigma'}(E,k). \tag{12}$$

Thus, we obtain a live many-body quasi-particle with a lifetime $\tau_k=1/\delta=\infty$ and an energy dispersion law determined by $\Delta_0(E,k)=0$, which reads

$$E_\sigma^*(k)=\frac{\hbar^2k^2}{2m^*}+\frac{\Sigma_+^+(k)+\Sigma_-^-(k)}{2}+\sigma\alpha_k^*k \tag{13}$$

with again $\sigma=\pm1$ and

$$\alpha_k^*=\left[\left(\alpha+\frac{\Sigma_+^+(k)-\Sigma_-^-(k)}{2k}\right)^2+\frac{\Sigma_+^-(k)\Sigma_-^+(k)}{k^2}\right]^{1/2}$$

being the effective Rashba parameter in the presence of e–e interaction. With the inclusion of e–e interaction, the electron density in the σ spin branch is determined by the diagonal elements of the DOS through

$$n_\sigma=\int_{-\infty}^{\infty}\mathrm{d}E\ f(E)D_{\sigma\sigma}(E) \tag{14}$$

with $f(x)$ being the Fermi–Dirac function, which gives, for $T\to0$,

$$n_\sigma=\frac{n_\mathrm{e}}{2}-\sigma\int_{k_\mathrm{F}^+}^{k_\mathrm{F}^-}\mathrm{d}k\frac{2\alpha k+\Sigma_+^+(k)-\Sigma_-^-(k)}{8\pi\alpha_k^*}. \tag{15}$$

Here k_F^σ is the solution for k resulting from $E_\sigma^*(k)=E_\mathrm{F}$ with E_F being the Fermi energy and

$$n_\mathrm{e}=n_++n_-=[(k_\mathrm{F}^+)^2+(k_\mathrm{F}^-)^2]/4\pi \tag{16}$$

is the total electron density. We have considered that a 2DEG is in equilibrium so that a single Fermi level is present in the system.

Comparing the energy spectrum in the presence of e–e interaction $E_\sigma^*(k)$ (see equation (13)) with that given by equation (2), we see that because $\Sigma_\sigma^\sigma(k)<0$, $E_\sigma^*(k)$ is always smaller than $E_\sigma(k)$ at a fixed k. This indicates that the e–e interaction can lower the energy of an electronic system, which is a well known consequence and has been observed in spin-degenerate systems [27]. The difference between $\Sigma_+^+(k)$ and $\Sigma_-^-(k)$ is relatively small. Together with the fact that $\Sigma_+^-(k)\Sigma_-^+(k)>0$, we immediately see from equation (13) that $\alpha^*>\alpha$. This implies that the e–e interaction enlarges the spin-splitting in a 2DEG with the Rashba SOI.

For a given total electron density n_e and a given Rashba parameter α, two Fermi wavevectors, $k_\mathrm{F}^\pm$, can be determined by solving equation (16) and an equation $E_+^*(k_\mathrm{F}^+)=E_-^*(k_\mathrm{F}^-)$, which gives a condition that a single Fermi level is across the system. The Fermi energy E_F can be calculated by $E_\mathrm{F}=E_+^*(k_\mathrm{F}^+)=E_-^*(k_\mathrm{F}^-)$ and the electron density in the σ spin branch by equation (15). Thus, the physical quantities, such as quasi-particle energy spectrum ($E_\sigma^*(k)$), DOS ($D_{\sigma\sigma'}(E)$), Fermi energy (E_F) and Fermi wavevector in different spin branches (k_F^σ), electron density in different spin branches (n_σ), etc, can be determined self-consistently.

In order to know the major contribution of the self-energy to spin-splitting of a 2DEG and simplify the analytical results, we can perform the small-k and small-q expansion with regarding to the self-energy. When $q \ll 1$, the inverse screening length becomes

$$K_+(q) = C_+/q \qquad \text{and} \qquad K_-(q) = C_-q^2,$$

where $C_+ = (e^2m^*/\sqrt{3}\hbar^2\kappa)[k_F^+ + k_F^- + k_\alpha \ln[(k_F^- - k_\alpha)/(k_F^+ + k_\alpha)]$ and $C_- = (e^2/8\alpha\kappa)[(1/k_F^+) - (1/k_F^-)]$. For intra-SO transition within the '+' or '−' spin branch, $K_+(q) \to +\infty$ when $q \to 0$. This is the reason why the Hartree interaction for intra-SO transition does not contribute to the self-energy. The self-energy now becomes

$$\Sigma_\sigma^\sigma(k) = -\frac{e^2}{\kappa}\left[k_F^\sigma - \sqrt{C_+}\arctan\left(\frac{k_F^\sigma}{\sqrt{C_+}}\right) + \frac{k^2}{2}\left(-\frac{k_F^\sigma}{2(C_+ + (k_F^\sigma)^2)} + \frac{C_+k_F^\sigma}{C_+ + (k_F^\sigma)^2}\right)\right], \quad (17)$$

and

$$\Sigma_{-\sigma}^\sigma(k) = -\frac{e^2}{\kappa}\left[\frac{1}{C_-}\ln(1 + C_-k_F^\sigma) - \frac{15}{16}k\right]. \quad (18)$$

In the self-consistent calculation, when $k \ll 1$, the k^2-term in equation (17) and the k-term in equation (18) are respectively much smaller than the k-independent terms. So in practical calculations in obtaining the physical properties such as the DOS, Fermi wavevectors, electron density in different spin channels, etc, we may neglect the k-dependent terms in equations (17) and (18). In doing so (i.e., taking $k = 0$), there is a k-dependent term in $\Sigma_\sigma^{\sigma'}(k)$, which implies that the e–e interaction can always affect the spin-splitting with the wavevector varying from 0 to $k_F^\pm$.

3. Results and discussion

In this paper, we consider a spin-split 2DEG on the basis of a InGaAs/InAlAs heterojunction with typical sample parameters such as the total electron density $n_e \sim 10^{11}$ cm^{-2} and the Rashba parameter $\alpha \sim 10^{-11}$ eV m. The material parameters corresponding to InGaAs are taken as follows: (i) the electron effective mass $m^* = 0.042m_e$ with m_e being the rest electron mass and (ii) the static dielectric constant $\kappa = 12.9$. Furthermore, we use the results obtained by Xu [26] for the calculation of the inverse screening length for different transition channels.

The dependence of the electron distribution in different spin branches on n_e and α is shown respectively in figures 1 and 2. Here we compare three groups of results obtained: (i) in the absence of e–e interaction, where the electron density in the $\pm$ spin channel is given simply as [23, 26]: $n_\pm = n_e/2 \mp (k_\alpha/2\pi)\sqrt{2\pi n_e - k_\alpha^2}$; (ii) in the presence of e–e interaction but under the small-k and small-q approximation; and (iii) in the presence of e–e interaction without small-k and q approximation. From these results, we see that the presence of the exchange interaction can enhance significantly the spin-splitting in a 2DEG over a wide regime of n_e and α. With increasing α or decreasing n_e, the electron density difference between different spin branches increases. This implies that a stronger spin polarization ($P = (n_- - n_+)/(n_- + n_+)$) occurs in a sample with a larger Rashba parameter α and/or a lower total electron density n_e. In addition, a full spin-polarization (i.e., $n_- = n_e$ and $n_+ = 0$) can be achieved via e–e interaction at a smaller α and/or larger n_e, in comparison with the case where the e–e interaction is absent. We also find that the results obtained with and without small-k and q approximation are roughly the same. It therefore suggests this to be a good approximation when looking into the effect of e–e interaction on spintronic properties.

To understand the physical reasons behind an exchange-enhanced spin-splitting in a 2DEG in the presence of the Rashba SOI, in figure 3 we show the energy dispersion relation at a fixed n_e and a fixed α for cases with and without inclusion of the e–e interaction. Here the energy

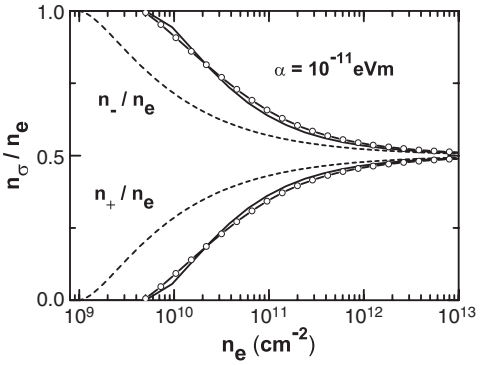

Figure 1. Electron distribution in different spin branches as a function of total electron density n_e at a fixed Rashba parameter α as indicated. The dashed curves are results obtained in the absence of e–e interaction. The solid curves (without small-k and q approximation) and symbols (under small-k and q approximation) are obtained in the presence of e–e interaction. Here n_σ is the electron density in the σ spin branch.

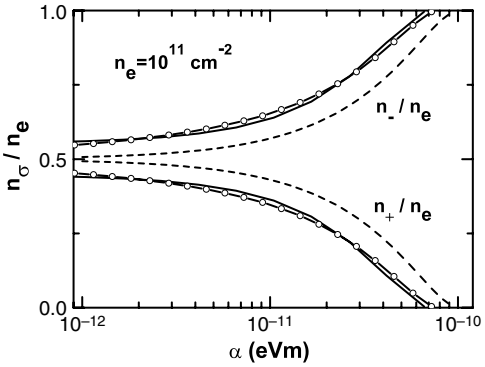

Figure 2. Electron density in different spin branches n_σ as a function of the Rashba parameter α at a fixed total electron density n_e for cases with (solid curves and -o- symbols) and without (dashed curves) e–e interaction. The -o- symbols and solid curves are results, respectively, with and without small-k and q approximation.

spectra ($E_\sigma(k)$ and $E^*_\sigma(k)$) are obtained by using equations (2) and (13) respectively for cases without and with e–e interaction. In the absence of e–e interaction, the Fermi level is [26] $E_F = (\hbar^2/m^*)(\pi n_e - k_\alpha^2)$. Firstly, we note that, similar to the case of a spin-degenerate electron gas, the exchange interaction lowers the energy and the Fermi level of the system. Secondly, the presence of the e–e interaction results in a split energy band even at $k = 0$, in sharp contrast to the case without e–e interaction where $E_+(0) = E_-(0) = 0$. It is known that small-k states are most likely occupied by electrons. An energy gap $E^*_+(0) - E^*_-(0) \neq 0$ implies that more DOS below E_F is induced in the '$-$' spin branch so that more electrons can stay in the '$-$' spin branch than in the '$+$' branch. This is one of the main reasons why spin-splitting can be enhanced by e–e interaction. Thirdly, we know that the intersections of the curves for $E_\pm(k)$ or $E^*_\pm(k)$ with E_F, projected onto the k-axis, give the Fermi wavevector $k_F^\pm$ for different spin branches. The difference $k_F^- - k_F^+$ leads to a difference in k-space area: $\pi(k_F^-)^2 > \pi(k_F^+)^2$. Accordingly, the electron densities in the $\pm$ branches are different. From figure 3, we see that the presence of the exchange interaction can significantly enlarge the difference between k_F^-

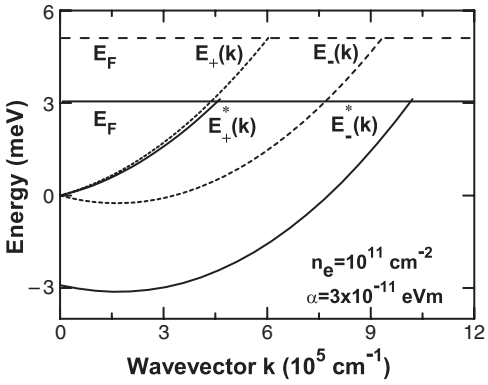

Figure 3. Dispersion relation for a spin-split 2DEG without (dashed curves and equation (2)) and with (solid curves and equation (13)) inclusion of e–e interaction for fixed electron density n_e and Rashba parameter α. E_F is the Fermi energy.

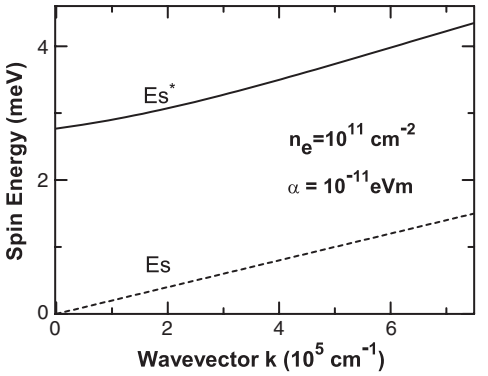

Figure 4. Spin energy as a function of electron wavevector k at the fixed total electron density and Rashba parameter. The results are shown for cases without (E_s and dashed curves) and with (E_s^* and solid curves) e–e interaction.

and k_F^+, and this becomes another major reason why the many-body effect can enhance spin-splitting in a 2DEG. With increasing α, the SOI in the system increases and the difference between n_- and n_+ increases. With decreasing n_e, the Fermi level of the system decreases so that more electrons are in the lower-energy '$-$' branch. As a result, the difference between n_- and n_+ increases with decreasing n_e. We see these interesting features clearly in figures 1 and 2.

To see more clearly the enhancement of spin-splitting by exchange interaction in a 2DEG on top of the Rashba effect, in figure 4 we compare the spin energy obtained with and without inclusion of e–e interaction. In the absence and presence of e–e interaction, the spin-energies are defined respectively as $E_s = 2\alpha k$ and $E_s^* = 2\alpha_k^* k$. As can be seen, the spin energy is much larger for the case where the e–e interaction is included. It is interesting to note that at a finite wavevector $k \sim 10^5$ cm^{-1} and using typical sample parameters such as $n_e \sim 10^{11}$ cm^{-2} and $\alpha \sim 10^{-11}$ eV m the spin energy with e–e interaction is about 10 times larger than that without e–e interaction. Due to the presence of the energy-gap at $k = 0$ induced by exchange interaction, a more pronounced enhancement of spin energy can be observed at smaller k values.

From a fundamental perspective, the e–e interaction in a 2DEG with the Rashba SOI has some unique features. In such a system, the spin orientation can change continuously with the momentum orientation when an electron moves in **k** space. We know that the exchange interaction is achieved mainly through altering the electron wavevector (or momentum). Thus, under the action of the SOI and the e–e interaction, the spectra of the $\pm$ spin branches can be shifted continuously in **k** space instead of a quantized spectrum in energy space for the usual case. More importantly, the lifting of the spin degeneracy in **k** space opens up new channels for electronic transitions. As a result, in the presence of e–e interaction the electrons are able to change their spin orientation simply through momentum exchange, which can be more easily achieved than that through energy exchange for the usual case. Consequently, a larger spin-splitting can be observed in a 2DEG in the presence of the Rashba SOI.

It should be noted that, as pointed out by Gerhardts and co-worker [20], the exchange interaction can enhance spin splitting in an electron gas system even in the absence of SOI. This has been verified experimentally by the spin-g-factor measurements in the presence of magnetic fields B (see experimental data cited by Gerhardts [20] and Xu *et al* [21]). It is well known that in the presence of quantizing magnetic fields the strong effect of the exchange interaction on Zeeman spin-splitting can be observed when the Fermi level is in between the spin-up and spin-down states for the same Landau level. In contrast, the exchange-enhanced spin-splitting in a 2DEG with the Rashba SOI can be observed at $B = 0$ when both spin branches are occupied by electrons (see figure 3). Instead of an increase in spin energy for Zeeman splitting with the magnetic field, the spin energy for Rashba spin-splitting in a 2DEG increases with electron wavevector $k < k_{\mathrm{F}}^{\pm}$ (see figure 4). Moreover, it has been demonstrated by Chen and co-worker [18] that in the presence of exchange interaction the enhancement of the Zeeman splitting is larger than that of the Rashba splitting, especially in low-density samples.

In the present study, we have developed a tractable theoretical approach in dealing with e–e interaction in a 2DEG in the presence of the Rashba effect. Comparing to a Fermi liquid theory proposed by Chen and co-worker [18], the Hartree–Fock approximation and the corresponding Green's function approach are more straightforward and transparent. On the basis of this approach, we can obtain important electronic properties such as the electron DOS, energy spectrum, Fermi wavevector in different spin branches, etc, and these properties can be calculated self-consistently. We also examine the influence of exchange interaction on spintronic properties such as spin energy and electron density in different spin branches, in conjunction with typical sample parameters realized from InGaAs/InAlAs heterojunctions. These results are more transparent and experimentally measurable than those obtained by Chen *et al*, where the factors of Zeeman and spin–orbit splitting were shown as a function of interaction parameter r_{s}. Noting that $r_{\mathrm{s}} = (a^*\sqrt{\pi n_{\mathrm{e}}})^{-1}$ increases with decreasing n_{e}, with $a^* = 4\pi\kappa\hbar^2/(e^2m^*)$ being the effective Bohr radius, the results obtained in the present study are in line with those shown in the reference. For example, Chen *et al* found that the enhancement of both the Zeeman and spin–orbit splitting increase with r_{s}. This is consistent with what is shown in figure 1 here, where the difference of electron density in different spin branches increases with decreasing n_{e}. Comparing our results with those obtained by Chen *et al*, we find that the exchange-enhanced Rashba spin-splitting obtained by the Hartree–Fock approximation is similar to that obtained by the Fermi-liquid theory with local-field correction. As pointed out by Chen *et al* [18], at present most of the spintronic devices realized from InGaAs/InAlAs heterojunctions have relatively high electron densities ($n_{\mathrm{e}} \sim 10^{11}$–$10^{12}$ cm^{-2}). This corresponds to $r_{\mathrm{s}} < 1$. Hence, the e–e interaction via exchange scattering plays an essential role in the enhancement of the spin-splitting in these device systems. Because both the Hartree–Fock approximation and the Fermi-liquid theory work well for the case of $r_{\mathrm{s}} < 1$, it is not surprising that two approaches can lead to similar results.

From the results obtained from the present theoretical study, we can draw an important conclusion that the many-body effects such as exchange interaction can enhance spin-splitting markedly in a Rashba spintronic system. In a 2DEG in which the Rashba SOI is present, the exchange interaction can play the following roles. (1) It can create more DOS for electrons to be in the lower-energy '$-$' spin channel. (2) It can lower the Fermi energy of the system. A low Fermi level pushes more electrons to the '$-$' spin branch, which has more DOS for electrons. (3) It can induce an energy gap between the $\pm$ spin branches even at $k = 0$ and can increase the gap in the $\pm$ energy states. (4) It can enlarge the difference of the Fermi wavevector in different spin branches. All these many-body effects favour increasing of spin-splitting of a 2DEG. As a result, we believe that exchange-enhanced spin-splitting is another important origin for which a strong Rashba SOI can be observed in InGaAs-based 2DEG systems.

4. Conclusions

In this paper, we have developed a simple and tractable theoretical approach to study many-body effects in a 2DEG in the presence of the Rashba spin–orbit interaction (SOI). This is based on a standard Hartree–Fock approximation and Green's function approach. From this theoretical model, we can calculate important spintronic properties (e.g. electron density of states, electron energy spectrum, electron density and Fermi wavevector in different spin branches, etc) self-consistently. We have examined the influence of e–e interaction on spin energy, electron energy spectrum, and electron distribution in different spin channels. The main conclusions drawn from this study are summarized as follows.

Similar to spin-degenerate electron gas systems, under the random-phase approximation the Hartree interaction does not affect the self-energy of a spin-split 2DEG. As a result, the self-energy induced by e–e interaction in a 2DEG with the Rashba SOI comes from exchange interaction alone. The presence of the exchange interaction can lower the energy spectrum and the Fermi level of a spintronic system and increase the difference of the Fermi wavevector in different spin branches. Together with an energy gap between the $\pm$ spin branches at $k = 0$, the exchange interaction can significantly enhance the spin-splitting of a 2DEG on top of the Rashba effect. By examining the influence of exchange interaction on spintronic properties such as spin energy, electron density in different spin channels, electron energy spectrum, etc, we have demonstrated that the electron–electron interaction is another important contribution for observation of strong Rashba spin-splitting in InGaAs-based 2DEG systems. We hope our theoretical findings can at least partly interpret why the values of α determined experimentally are larger than those obtained theoretically using the $\mathbf{k}\cdot\mathbf{p}$ calculation (single-particle approach) and the self-consistent calculation on the basis of the $\mathbf{k}\cdot\mathbf{p}$ results (exchange interaction is not included).

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References

[1] For a recent review, see, e.g. Zawadzki W and Pfeffer P 2004 *Semicond. Sci. Technol.* **19** R1
[2] Schapers Th, Engels G, Lange J, Klocke Th, Hollfelder M and Luth H 1998 *J. Appl. Phys.* **83** 4324
[3] Rashba E I 1960 *Sov. Phys.—Solid State* **2** 1109

Rashba E I and Sheka V I 1991 *Landau Level Spectroscopy* vol 1, ed G Landwehr and E I Rashba (Amsterdam: North-Holland) p 131
[4] Datta S and Das B 1990 *Appl. Phys. Lett.* **56** 665
[5] Koga T, Nitta J, Takayanagi H and Datta S 2002 *Phys. Rev. Lett.* **88** 126601
[6] Wang X F, Vasilopoulos P and Peeters F M 2002 *Phys. Rev.* B **65** 165217
[7] Sato Y, Kita T, Gozu S and Yamada S 2001 *J. Appl. Phys.* **89** 8017
[8] Schapers Th, Engels G, Lange J, Klocke Th, Hollfelder M and Luth H 1998 *J. Appl. Phys.* **83** 4324
[9] Lin Y P, Koga T and Nitta J 2005 *Phys. Rev.* B **71** 045328
[10] Pfeffer P and Zawadzki W 1999 *Phys. Rev.* B **59** R5321
[11] Schapers Th, Knobbe J, van der Hart A and Hardtdegen H 2003 *Sci. Technol. Adv. Mater.* **4** 19
[12] Yamada S, Sato Y, Gozu S and Kikutani T 2000 *Physica* E **6** 771
[13] Kita T, Sato Y, Gozu S and Yamada S 2001 *Physica* B **298** 65
[14] Koga T, Nitta J and Datta S 2003 *Physica* E **18** 161
[15] Das B, Datta S and Reifenberger R 1990 *Phys. Rev.* B **41** 8278
[16] Grundler D 2000 *Phys. Rev. Lett.* **84** 6074
[17] Malcher F, Lommer G and Rossler U 1986 *Superlatt. Microstruct.* **2** 267
[18] Chen G H and Raikh M E 1999 *Phys. Rev.* B **60** 4826
[19] Ando T, Fowler A B and Stern F 1982 *Rev. Mod. Phys.* **54** 437
[20] Manolescu A and Gerhardts R R 1995 *Phys. Rev.* B **51** 1703
[21] Xu W, Vasilopoulos P, Das M P and Peeters F M 1995 *J. Phys.: Condens. Matter* **7** 4419
[22] See, e.g. Mattuck R D 1976 *A Guide to Feynman Diagrams in the Many-Body Problem* (New York: McGraw-Hill)
[23] Xu W 2003 *Appl. Phys. Lett.* **82** 724
[24] Chen G H and Raikh M E 1999 *Phys. Rev.* B **59** 5090
[25] Mishchenko E G and Halperin B I 2003 *Phys. Rev.* B **68** 045317
[26] Xu W 2005 *Phys. Rev.* B **71** 245304
[27] See, e.g. Devreese J T, Kunz A B and Collins T C (ed) 1974 *Elementary Excitations in Solid, Molecules and Atoms* (New York: Plenum)